FACULDADE DE CIÊNCIAS DA UNIVERSIDADE DE LISBOA

ANÁLISE ASSIMPTÓTICA E APLICAÇÕES A PROBLEMAS DA MECÂNICA DO CONTÍNUO

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SYNOPSIS

In this thesis we present applications of the Theory of Homogenization to three distinct problems : the first one is related to the convergence of an algorithm in shape optimization ; the second one is about characterizing the Γ-limit corresponding to a class of problems presenting nonlocal assymptotic behaviour in time ; the third problem is related to a computational study of Young measures. Original, nonclassical results are obtained.

The presentation in divided in four chapters :

Chapter I presents shortly results of Convex Analysis, Measure Theory and Homogenization, which are used in the subsequent chapters.

Chapter II is entirely devoted to the justification of computational methods that are in use, namely to the proof of the convergence of an algorithm which is empirically used in shape optimization of elasticity problems by G. Allaire and G. Francfort (see [AF] and [ABJF]). We study an optimization problem in thermal conductivity. The initial problem is reformulated in terms of heat flux (Section II.1) and this reformulation is shown to be ill-posed. The problem is therefore relaxed in two ways, one using homogenization theory, another one using classical relaxation techniques (Section II.2). Results obtained by G. Allaire, L. Tartar and F. Murat are used. Section II.3 presents a result on minimizing sequences for the relaxed formulations of the problem ; this result was stated in [To1] and is essential to the understanding of the algorithm. The main result of this chapter is Theorem II.4.2, stated in [To2], about the convergence of the algorithm (Section II.4). Another version of the same result, more complete and more appropriate for practical approaches, was presented in [To3]. These results legitimate the use of the above mentioned algorithm in the conductivity case or in other equivalent frameworks. However, in elasticity, this problem is still open.

In Chapter III we describe the notion of scale convergence, which generalizes the two-scale convergence to the nonperiodic case. The scale convergence, used in [M3] and [To4] for the study of a class of problems involving nonlocal effects in time, has been developped afterwards in [MTo]. As in the periodic case, in order to describe the assymptotic behaviour of an oscillating problem, it is reasonable to consider, among all possible oscillations, only those that synchronize with the given oscillating coefficients of the problem. This is the basic idea of scale convergence, whose definition is presented in Section III.1, together with some related results, also stated in [MTo]. The main result is the compactness theorem (Theorem III.1.2) whose proof uses techniques of Young measures. Section III.2 deals with examples and applications of the scale convergence. In the first place, we show that known techniques like the two-scale convergence, the multi-scale convergence and the θ-convergence are particular cases of scale-convergence (see Subsections III.2.1 and III.2.2). We systematize the results about the characterization of the Γ-limit of functionals involving nonperiodic oscillations, both time-independent and time-dependent (see Subsections III.2.3 and III.2.4). The main results are Theorems III.2.6 and III.2.11 which can also be found in [MTo]. We consider the example of a parametrized ordinary differential equation with oscillating coefficients, studied by L. Tartar (see [T2] and [T3]) and M. L. Mascarenhas (see [M1], [M2] and [M3]) ; we study a generalization of this example for time dependent coefficients ; our approach is based on Γ-convergence (see Subsection III.2.5). The results obtained for the time-dependent case can also be found in [To4]. When the coefficients of the equation are analytic in time, the expression of the Γ-limit of the associated energy functionals turns out to be simpler. This particular case is of practical importance, as it can be seen as a mathematical model of a laminate made of two materials whose coefficients are analytical in time. Directions of future developments include the adaption of scale convergence to problems involving more complex differential operators or concentrations : the technical details are an open problem.

In Chapter IV we propose a computational method for approximating Young measures. This method, inspired by the Monte-Carlo method, is based on the very definition of a Young measure associated to a sequence. It allows one to approximate weak limits of sequences, as well as moments of Young measures. Theorem IV.1.1 and Proposition IV.1.5 justify the method. However, the described method still contains empirical elements. Three examples are presented in which we compare theoretical and numerical results (see Section IV.3). This method, as well as some of the numerical experiments, are the subject of the last section of [MTo]. The proposed techniques allow a future numerical study of nonlocal effects and memory effects induced by homogenization, since the mathematical expression of such effects involves Young measures.

SUMÁRIO

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